STABILITY OF THE MOTION OF A SOLID PARTICLE IN A PLANAR ROTATING FLOW

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The effect of the shape of a rotating flow on the motion of a particle in it is studied. The conditions for and regions of stable motion of a solid particle are found.

In several studies [1-3] of dust separation in a planar rotating flow with a vertical axis and central shaft, it has been shown that an "equilibrium circle" exists for every dust particle. When a dust particle moves uniformly along this circle, the oppositely directed centrifugal force F_c and drag force F_d are equal in magnitude. The equation for equilibrium motion for the particle along the circle is therefore

$$F_c = F_d,$$

or, in expanded form,

$$\rho_2 \frac{\pi\delta^3}{6} \frac{v_\varphi^2}{r} = \psi \frac{\pi\delta^2}{4} \frac{\rho_1 v_r^2}{2}. \quad (1)$$

Let us consider the stability conditions for this equilibrium. We convert Eq. (1) to dimensionless form:

$$\mathrm{ctg}^2\alpha\, \mathrm{ctg}\,\alpha_0\, \mathrm{Re}^3/C = \psi\, \mathrm{Re}^2, \quad (2)$$

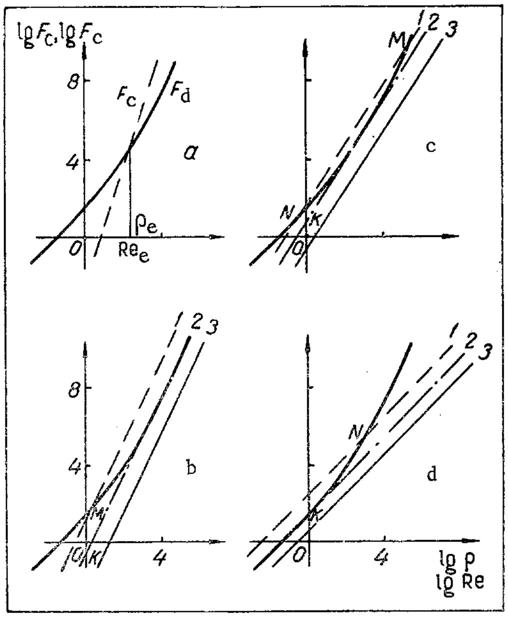

Fig. 1. Regions of stable-equilibrium motion of the particle. a) n = 1.0; b) 0.5; c) 0.25; d) 0. [1) $A > A_{cr}$; 2) $A = A_{cr}$; 3) $A < A_{cr}$].

where $\mathrm{Re} = \delta v_r \rho_1/\eta$ is the Reynolds number for a particle moving along the circle, $\cot\alpha = v_\varphi/v_r$; $\cot\alpha_0 = v_\varphi/v_{r_0}$; $C = 3v_{\varphi_0} r_0 \rho_1^2 / 4\eta\rho_2$. Since the continuity equation for planar flow is $v_r r$ = const, a dependence $\cot\alpha = f_1(r) = f_2(v_r) = f_3(\mathrm{Re})$ can always be found experimentally in the most general case; then, using the notation $\cot\alpha_0/C = B$, we rewrite Eq. (2) as

$$F_c = B\xi(\mathrm{Re}) = F_d = \psi\,\mathrm{Re}^2. \quad (3)$$

The roots of (3) are the Reynolds numbers Re_e for equilibrium trajectories, along which the dimensionless trajectory radii can be calculated:

$$\rho_e = r_e/r_0 = R\,\mathrm{ctg}\,\alpha_0/\mathrm{Re}_e \quad (R = \delta v_{\varphi_0}\rho_1/\eta).$$

The $\psi\mathrm{Re}^2 = \varphi_1(\mathrm{Re})$ dependence is of completely definite form for dust particles of known shape, and it can be calculated from experimental data for, e.g., spherical particles [4]. The form of the $B\xi(\mathrm{Re}) = \varphi_2(\mathrm{Re})$ curve is governed by the form of the function $\xi(\mathrm{Re})$ and the quantity B, which affect the number of roots of (3), i.e., the number of equilibrium trajectories; they also affect the form of the equilibrium if there is one.

If the $F_c = \varphi_2(\mathrm{Re})$ curve intersects the $F_d = \varphi_1(\mathrm{Re})$ curve from the left and from below, i.e., $dF_c/d\mathrm{Re} > dF_d/d\mathrm{Re}$

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TABLE 1. Characteristic Parameters for the Trajectories of Fig. 2 ($\cot \alpha_0 = 1$)

n	A_{cr}	Trajectory No.	C	R	A	ρ_e^{st}	ρ_e^{un}	ρ_{min}
0,5	0,38	1	100	140	1,4	2,25	—	0,485
		2	100	80	0,8	0,407	—	0,402
		3	342	82	0,24	—	—	0,000
0,25	10,0	4	640	1280	71,6	$1{,}5 \cdot 10^4$	$3{,}4 \cdot 10^{-2}$	0,505
		5	640	640	25,3	112,0	0,142	0,468
		6	640	320	8,95	—	—	0,000
0,00	22,4	7	128	230	415	—	0,262	0,456
		8	128	179	323	—	0,422	0,422
		9	128	76,8	46	—	6,30	0,000
		10	128	384	11,5	—	—	0,000

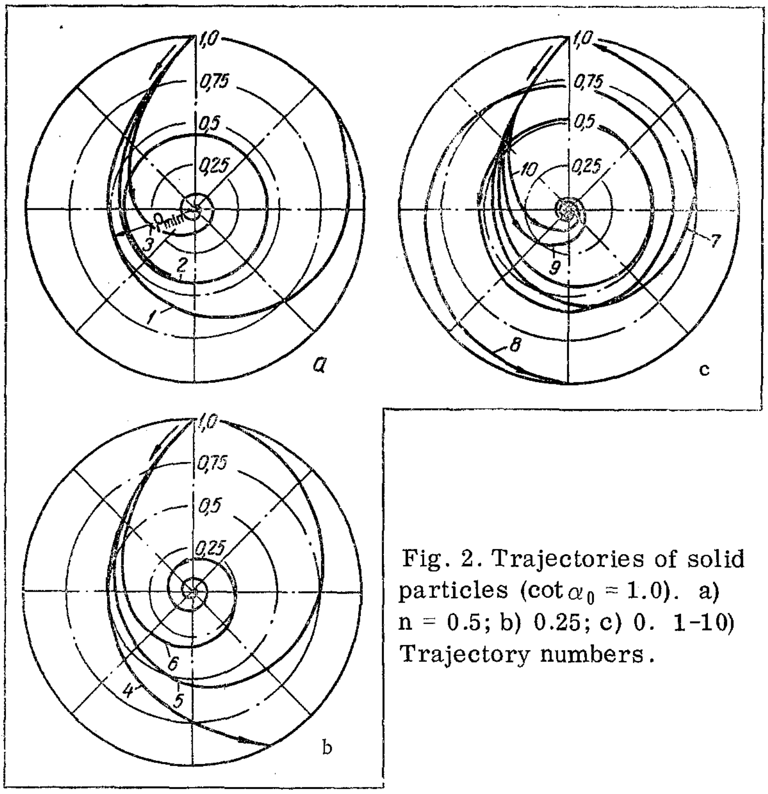

Fig. 2. Trajectories of solid particles ($\cot \alpha_0 = 1.0$). a) n = 0.5; b) 0.25; c) 0. 1-10) Trajectory numbers.

with $Re = Re_e$, the equilibrium rotation of the particle is stable. If, because of some external perturbation, the particle is shifted to a radius $\rho' < \rho_e$ ($Re' > Re_e$), then we have $F_c' > F_d'$, and the particle tends to return to the previous trajectory of radius ρ_e. Analogously, if $\rho' > \rho_e$, we have $F_c' < F_d'$, and the particle tends to return to $\rho = \rho_e$.

On the other hand, if the F_c curve intersects the F_d curve from the left and from above ($dF_c/dRe < dF_d/dRe$ with $Re = Re_e$), the equilibrium is unstable. If these curves do not intersect, there is no equilibrium trajectory.

Let us consider the particular case in which the radial profile of the tangential velocity is approximated by $v_\varphi r^n = \text{const}$, where n varies from -1 (quasirigid rotation) to 1 (potential flow). Then Eq. (3) can be written

$$A\,\mathrm{Re}^{1+2n} = \psi\,\mathrm{Re}^2, \tag{4}$$

where

$$A = R^{2-2n}\,\mathrm{ctg}^{2n+1}\alpha_0/C.$$

The existence and type of equilibrium are governed by the quantities n and A. We limit the discussion to the range $Re \leq 10^5$, corresponding to conditions for separation of real dust. Here the $\log F_d = f(\log Re)$ dependence for $0 < Re \leq 0.1$ ($\psi = 24/Re$) is described by a straight line with a slope of one; for $0.1 < Re \leq 10^3$, it is described by a concave curve; for $10^3 < Re \leq 10^5$ ($\psi \approx 0.4$), it is described by a straight line with a slope of two. In Fig. 1, the curves show the $\log F_d = f(\log Re)$ dependence, while the lines 1-3 show the $\log F_c = f(\log Re)$ dependence for various A in Eq. (4).

It follows (Fig. 1) that: 1) there is one stable-equilibrium position for all A when $0.5 < n \leq 1$ (Fig. 1a); 2) when $n = 0.5$, there exists either one stable-equilibrium position at point M ($A > A_{cr}$), or there is no equilibrium ($A \leq A_{cr}$) (Fig. 1b); 3) when $0 < n < 0.5$, there are two equilibrium positions if $A > A_{cr}$ (stable at small Re – point N – and unstable at large Re – point M), there is one unstable equilibrium ($A = A_{cr}$), or there is no equilibrium $A < A_{cr}$ (Fig. 1c); 4) when $n = 0$, there is either one unstable equilibrium at N ($A > A_{cr}$) or there is no equilibrium ($A \leq A_{cr}$) (Fig. 1d); 5) when $n < 0$, there is a single unstable equilibrium for all A. The value of A_{cr} (the segment 0K in Fig. 1b-1d) increases with decreasing n (Table 1).

As an example of this procedure, we have used a Ural-2 computer to solve the system of dimensionless differential equations of motion of the particle [3] for several parameter values with the following initial conditions: $\varphi_0 = 0$; $\rho_0 = 1$; $W_{\varphi_0} = 0.01$, and $W_{r_0} = -0.01$. The calculated results are shown in Table 1 and in Fig. 2.

When $0 < n \leq 0.5$ with $A < A_{cr}$, and when $n \leq 0$ with any A, the particle tends to move either toward the rotation center ($\rho \to 0$) or toward the periphery ($\rho \to \infty$), depending on the position of the point at which the particle is introduced with respect to its equilibrium trajectory. If, e.g., we set $\rho_2/\rho_1 = 1000$, $r_0 = 1$ m, $\cot\alpha_0 = 1.0$, and $v_{\varphi_0} = 10$ m/sec; the A_{cr} value corresponds to $\delta_{cr} = 160\text{-}440\ \mu$ for various n.

Accordingly, if the limiting size for the dust separation satisfies $\delta_b < \delta_{cr}$ (as is true for most practical cases), centrifugal separation is impossible with $n \leq 0.5$.

NOTATION

δ, ρ_2	are the size and density of the particle;
η, ρ_1	are the dynamical viscosity and gas density;
ψ	is the drag coefficient;
r, v_r, v_φ	are the instantaneous values of the radius vector of the particle and the projections of the flow velocity on the radius vector and on the normal to it, respectively;
r_0, v_{r_0}, v_{φ_0}	are the characteristic values of the same quantities (at the points at which the particle is introduced);
C, R, B, A, Re	are the dimensionless characteristic parameters.

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